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ANALYSIS OF DEBRIS FLOW SURGES USING THE THEORY OF UNIFORMLY PROGRESSIVE FLOW

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ABSTRACT

Coarse debris flows develop surges with distinct longitudinal sorting. Although highly unsteady, such flow often appears to attain a steady-state condition, moving over long distances with approximately constant velocity and maximum depth. Typically, a steep, bouldery front is followed by an accumulation of liquid slurry, which in turn decays into a dilute tail. Such sorting has long been recognized by field workers, but its influence on the dynamic behaviour of debris flow surges has not yet been fully clarified by analysis. A simple model is presented, using the theory of uniformly progressive flow and incorporating zoned longitudinal variation in rheology. It is shown that non-homogeneity can cause very significant magnification of the peak discharge, depending on the slope angle and on the length of the frontal boulder concentration. The shape of the surge flow profiles is determined not only by the rheology of the retained material, but by the longitudinal variation of material characteristics. As a result, excessive reliance on laboratory-derived rheological constitutive relationships is not advisable. Models of debris flow surges should be non-homogeneous and able to incorporate zones of contrasting rheology. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: debris flow; dynamics; flow behaviour

INTRODUCTION

Debris flows are characterized by highly unsteady, surging flow behaviour. Hydrodynamic modelling of debris flow surges is much more difficult than that of water flows. The main reason is the diversity of substances involved in debris flow, which include water, grain dispersions, mixtures of colloidal and granular particles in water and large solid particles such as boulders and timber. Many field observations attest to the heterogeneity of debris surges (e.g. Sharp and Nobles, 1953; Pierson, 1986).

As described by Pierson (1986), a typical surge has a steep front or 'head' with the densest slurry, the highest concentration of boulders and the greatest depth. This is followed by a progressively more dilute and shallower 'tail' (Figure 1). Compared to normal water floods, debris flow surges rise quickly and have short durations. Surging behaviour is present in both fine-grained debris flows and in coarse flows derived from strong rocks with low contents of colloidal particles and silt. A typical coarse debris flow surge front is pictured in Figure 2.

Surges in finer, more homogeneous debris flows, such as the 'turbulent mud flow' described by Takahashi (1991), may be shaped by the change in flow regime from turbulent in the front to laminar behind it (Davies, 1986; Hunt, 1994). The shapes of the coarser debris flow surges are controlled by their longitudinal heterogeneity and by the existence of boulder fronts (Pierson, 1986; Iverson, 1997a,b).

The purpose of this paper is to present a simple analytical model which can help explain the process of formation of coarse-grained debris flow surges and their characteristic longitudinal profiles.

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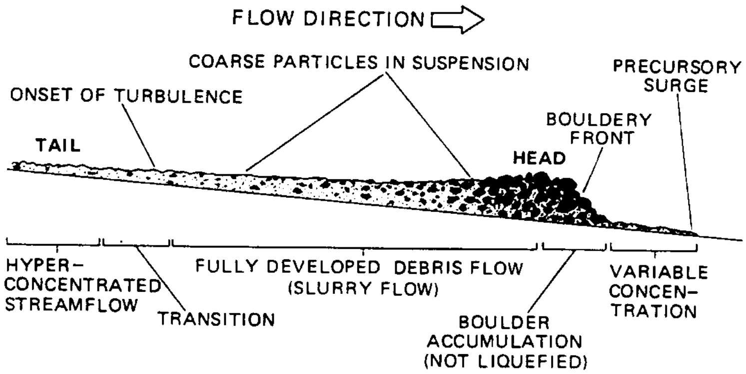

Figure 1. Schematic representation of a heterogeneous debris flow surge. Vertical scale exaggerated (Pierson, 1986)

Figure 2. A typical boulder front in a moving debris flow surge at Mount St Helens, Washington, USA (photo by T.C. Pierson, US Geological Survey)

DYNAMIC MODELLING AND RHEOLOGICAL CHARACTER OF DEBRIS FLOW

Homogeneous models

Dynamic modelling of debris flow has received considerable attention from researchers over the last few decades. The first models were concerned with steady uniform flow and its dependence on material rheology. One group of workers examined the Bingham model which appears suitable for relatively fine-grained debris flows including a certain fraction of clay (e.g. Yano and Daido, 1965; Johnson, 1970; Johnson and Rodine, 1984). Another group concentrated on the dilatant grain flow rheology, suitable for coarser debris flows (Bagnold, 1968; Breitfuss and Scheidegger, 1974; Takahashi, 1980, 1991). The use of the laminar Newtonian flow equations has been suggested for approximate analysis (Sharp and Nobles, 1953; Hungr *et al.*, 1984).

More recently, non-uniform and unsteady flow models have appeared, using rheologies belonging to one of the above groups, or more complex ones based on theory, laboratory viscosimeter tests or back-calculation (e.g. Chen, 1987; O'Brien *et al.*, 1993; Laigle and Coussot, 1993; Rickenman and Koch, 1997).

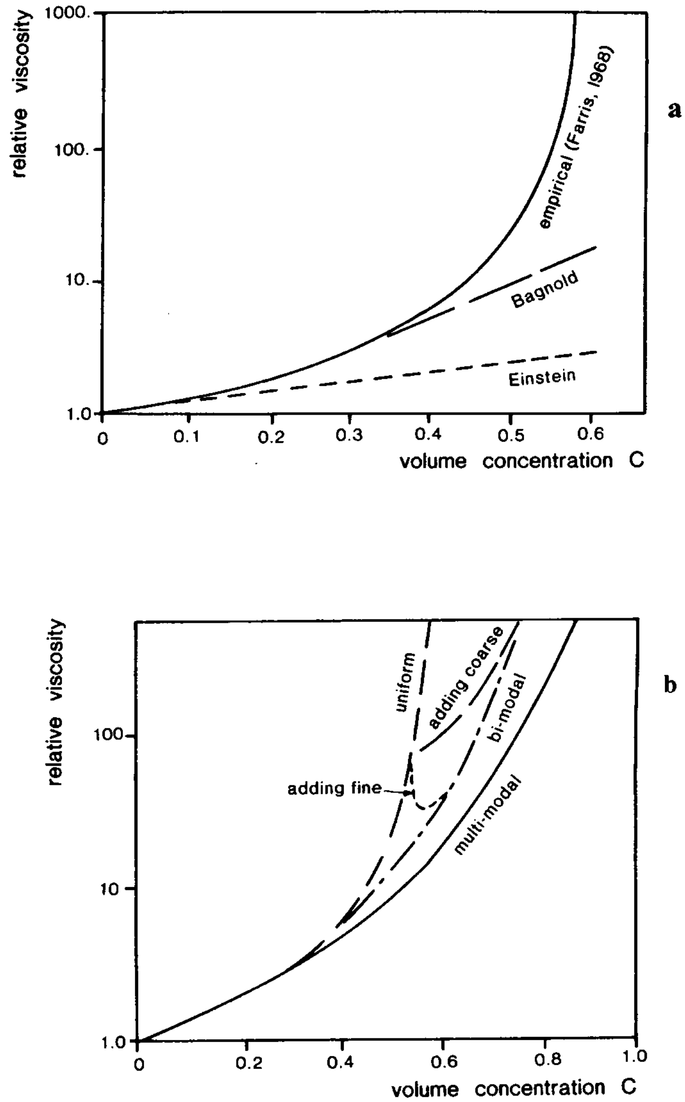

Figure 3. Relative viscosity of uniform (a) and graded (multi-modal) (b) grain dispersions in water, with concentrations corresponding to the macro-viscous regime (Farris, 1968)

Debris flow rheology

All of the work cited above is based on the assumption that debris flow involves homogeneous material. However, it has long been understood that debris flows are not homogeneous. Visual observations in the field by several authors revealed that both the overall solids concentration (and water content) and the concentration of large clasts vary with position within the body of a surge. One often-cited symptom of such variation is the calm, apparently laminar flow regime near the peak of a surge, contrasting with the turbulence of the intersurge flow (e.g. Pierson, 1980). Direct measurements of heterogeneity are rare. Takahashi (1991, p. 24) documented a decrease in the coarse clast content with distance behind the surge front. Pierson (1985, p. 138) measured a significant gradual reduction of solids concentration with distance behind the front. The origin of the longitudinal heterogeneity probably lies in the well known sorting mechanism, by which the larger particles accumulate at the upper flow surface and are then accelerated forward by a vertical velocity

gradient (Takahashi, 1991). It may be expected that the longitudinal sorting requires a certain period to 'mature' and that it changes in time due to selective deposition of coarse clasts.

Sediment concentration is known to strongly affect the rheology of granular mixtures (e.g. Pierson and Costa, 1987). For example, the bulk viscosity of a uniform mixture of sand grains and water tested by Farris (1968) varies by a factor of 10 for a volume concentration increase of only 6 per cent (0·55 to 0·58; see Figure 3a). Further increase of concentration leads to an asymptotic growth of viscosity, as the mixture gradually acquires measurable frictional strength (cf. Bagnold, 1954). Graded mixtures are also very sensitive to solids concentration, although somewhat less so than uniform dispersions (Figure 3b). Similar observations were reported by Major and Pierson (1992) for mixtures of sand, silt and clay, resembling the composition of fine debris flow slurries.

The rheology of graded mixtures containing large particles (gravel and coarser) cannot be studied in laboratory tests and may depend on grain size distribution. Iverson (1997a) obtained independent measurements of the total stress and pore pressure at the bed of experimental debris flows. The pore pressure was zero beneath the leading edge of the surge suggesting frictional flow, but rose to nearly the full value of the total stress behind the front, indicating the onset of liquefaction. The concept of effective stress is difficult to apply to debris flow material, as the distinction between fluid and solid constituents is not clear. Under static conditions, the pore fluid is water. During and after mixing, the coarse particles may be buoyed up by a heavy viscous slurry of the fine fractions (e.g. Major and Pierson, 1992). The dividing line between fluid and solid components may be dependent on strain rate and solids concentration.

Thus, the material forming a granular debris flow surge may well comprise a frictional fluid with variable pore pressure, a macroscopically viscous fluid with variable viscosity, or a transitional fluid (Bagnold, 1954). The presence of colloidal fractions may introduce yield strength (Major and Pierson, 1992). The flow regime may be laminar or turbulent depending on velocity, depth, solids concentration and grain size distribution.

Non-homogeneous models

Despite the fact that the existence of boulder fronts and other forms of non-homogeneity in debris flow surges has long been recognized by field workers, only a few non-homogeneous debris flow models have appeared to date. Takahashi (1980, 1991) developed theory to explain the concentration of boulders in a surge front. Hungr (1981) examined the dynamic equilibrium of a boulder front acting as a 'moving dam' and assumed arbitrarily that the frictional front is uplifted by a matrix fluid pressure varying linearly from zero at the tip of the surge to the full geostatic value at the surge peak. Iverson (1997a,b) measured a similar pore fluid pressure distribution in a large-scale flume experiment. He also adapted a frictional unsteady flow model to consider linearly varying pore pressure in a partially drained boulder front and was successful in predicting the velocity of propagation of an experimental debris flow surge.

On another front, Davies (1986), Hunt (1994) and Takahashi (1991) pointed out the possibility of turbulent surge flow heads supporting roll waves of laminar flow in fine-grained debris flows.

The importance of peak discharge

One of the most important characteristics of debris flow is its high peak discharge. It is the chief cause of the great destructiveness of debris flow surges. Peak discharge is perhaps the best parameter to distinguish a debris flow from a debris flood (hyperconcentrated flow). Both are capable of transporting large quantities of poorly sorted sediment. However, the movement of debris flood surges requires the tractive forces of flowing water. Thus, their peak discharge exceeds that of a clear water flood only by a bulking factor, generally less than two (Costa, 1984). In contrast, debris flow surges decouple from water flow and move independently under gravity. In the process of building a boulder front or a turbulent snout, a surge contracts in length and becomes increasingly steep-limbed (Pierson, 1986). The short-lived peak discharge can be as much as 40 times greater than that of an extreme flood (VanDine, 1985).

Estimation of the peak discharge is of vital importance in practical debris flow studies, as it determines the maximum velocity and flow depth, momentum, impact forces, ability to overrun channel walls and barriers,

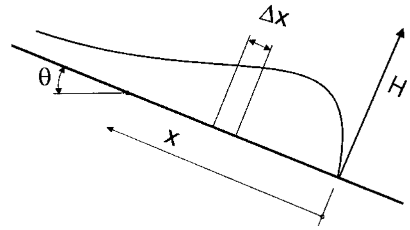

Figure 4. Reference framework for analysis of a debris flow surge as uniformly progressive flow

as well as the runout distance. The analysis presented below shows that longitudinal heterogeneity is a key factor in determining the peak discharge of coarse-grained debris flow surges.

APPLICATION OF THE UNIFORMLY PROGRESSIVE FLOW CONCEPT

Dynamic analysis of debris flow is greatly complicated by its unsteady and non-uniform character. However, surges travelling over long distances can be expected to approach a certain steady-state condition characterized by a fairly constant drop-shaped longitudinal profile. Mudline observations along debris flow paths often indicate nearly constant maximum flow cross-section area over long distances. Takahashi (1991, p. 7) shows velocity records of several debris surges. One varies between 2·9 and 3·9 m s^{-1} over a distance of 1000 m and another between 3·1 and 4·1 m s^{-1} over 1200 m, despite the presence of slope variations and check dams within the reaches in question. Video footages of debris flow surges in motion, such as those recorded at the Jiang Jia Ravine debris flow observation station in China, show approximately constant maximum depth and overall length of each individual surge over a period of time (Costa and Williams, 1984). Small-scale laboratory flume models with moving conveyor belt bases have been constructed, in which steady-state surges of water and grains, or slurry mixtures can be maintained (Hirano and Iwamoto, 1981; Davies, 1990).

The precise steady-state condition is probably rare in natural debris flows. Rather, fluctuations around the steady state, repeated shortening or elongation of the surge in response to various local irregularities, are probably the norm. This does not detract from the validity of the qualitative findings discussed below.

The approximate steady-state condition of some debris surges provides an opportunity for relatively simple analytical treatment, based on the concept of uniformly progressive flow. The basis of the mathematical approach is provided by St Venant's equation of unsteady flow in a prismatic shallow open channel. This can be derived by considering an average slice of the flow, H deep, whose width is Δx in the downslope direction and w perpendicular to it (see Figure 4). An application of the momentum conservation principle to this columnar element parallel to the flow base (bed), results in (e.g. Chow, 1959, p. 527):

$$\rho H w \Delta x\left(\frac{\mathrm{d}V}{\mathrm{d}t} + \beta V \frac{\mathrm{d}V}{\mathrm{d}x}\right) = \rho g H w \Delta x \sin\theta - T w \Delta x - \frac{\mathrm{d}p}{\mathrm{d}x} w \Delta x \tag{1}$$

where, x is a longitudinal coordinate referenced to a fixed origin, V is the mean velocity, θ is the bed slope angle, β is a momentum averaging correction factor, ρ is the flow density, T is the bed shear stress and $\mathrm{d}p/\mathrm{d}x$ is the longitudinal gradient of the hydrostatic thrust force. The term on the left-hand side of the equation is the inertial force, calculated as the mass of the slice multiplied by the sum of the local and convective accelerations. The terms on the right include the tangential component of gravity, the frictional (or velocity-dependent) resisting force and the lateral pressure resultant.

Iverson (1997a) derived the same equation more rigorously by an integration of the governing equations of mixture theory. He also proposed, following Savage and Hutter (1989), that the last term on the right-hand side of Equation 1 should allow for internal pressure anisotropy similar to the Rankine stress states, as appropriate for quasi-static deformation of a frictional mass. Hungr (1995) used the same concept to allow for

plastic deformation within the body of a rock or debris avalanche, moving as a relatively rigid plug over a thin fluid basal layer.

Under such conditions, it is reasonable to expect the pressure term to be dependent on the longitudinal strain in the moderately deforming landslide mass above the mobile layer. When the mass is being compressed, e.g. in a concave segment of the path, the stress tensor will approach the passive Rankine condition, with the longitudinal stress component exceeding the normal one perpendicular to the bed. In a region of extension, the opposite will be true. At the steady state of dynamic equilibrium, however, neither compression nor extension is occurring. The longitudinal strain in the granular mass is zero and the stress state should conform to an 'at rest' ($k_0 \sim 1{\cdot}0$) condition intermediate between the passive and active states. In the absence of direct measurements of internal stress in the debris flow front, it appears reasonable to accept the isotropic stress state implied in the last term of Equation 1. With detailed field observations of boundary stresses of the kind reported by Iverson (1997a), it should soon be possible to check the above assumption using the theory developed here, or a similar one.

Assuming, therefore, that the longitudinal pressure in the fluid equals the normal pressure and that the equipotential lines of the flow are perpendicular to the bed, the thrust force gradient can be expressed as (see also equation 17 in Hungr, 1995):

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\rho g H \frac{\mathrm{d}H}{\mathrm{d}x}\cos\theta \tag{2}$$

The Lagrangian form of St Venant's equation is referenced to moving longitudinal coordinates (also called x) increasing in the upstream direction and whose origin travels with the element under consideration as shown in Figure 4 ($w\Delta x$ cancels out in the equation):

$$\rho H \frac{\mathrm{d}V}{\mathrm{d}t} = \rho g H \sin\theta - T - \frac{\mathrm{d}p}{\mathrm{d}x} \tag{3}$$

The advantage of this form is that the convective acceleration, describing the longitudinal momentum flux along the fixed coordinate, disappears from the equation.

Uniformly progressive flow is defined as a condition in which the velocity in any moving element remains constant in time, i.e. $\mathrm{d}V/\mathrm{d}t = 0$ (Chow, 1959, p. 531). This assumption can be used to describe a translating surge which travels down the channel at a constant velocity, without changing its longitudinal profile. Under these conditions and with substitution from Equation 2, Equation 3 can be rearranged as:

$$\frac{\mathrm{d}H}{\mathrm{d}x}\cos\theta = \frac{T}{\rho g H} - \sin\theta = S_f - S \tag{4}$$

This equation describes the depth profile of a steady-state surge moving at a uniform velocity. The term T is the resisting shear stress on the base, which includes all the flow resistances depending on the rheology and regime of the flow. S is the bed slope and S_{f} is the friction slope.

APPLICATIONS OF THE THEORY

Homogeneous surge fronts

Equation 4 can be solved easily when the flow is homogeneous and T (or S_{f}) a unique function of velocity, V, depth, H, and some rheological parameters. For example, in case of a purely plastic fluid flowing in a wide channel, $T = \tau_{\mathrm{f}}$, the constant yield strength of the fluid, and an integration of Equation 4 yields:

$$\frac{xS}{H_n} = \frac{H}{H_n} + \ln(1 - \frac{H}{H_n}) \tag{5}$$

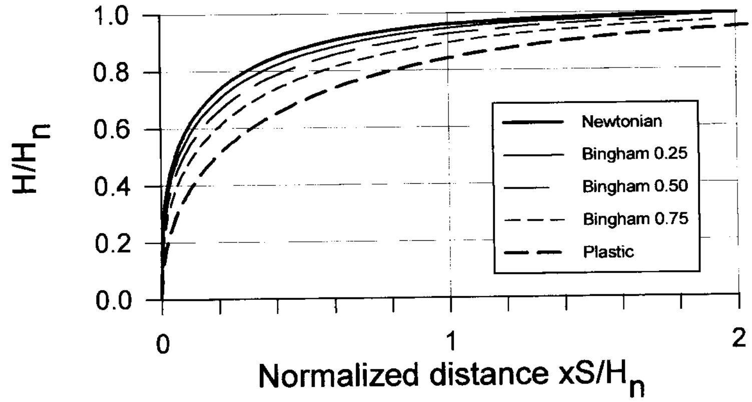

Figure 5. Dimensionless profiles of roll waves in homogeneous materials with different rheologies. H = flow depth, measured normal to the sloping bed; H_n = normal depth. The Bingham solutions are distinguished by the parameter N (see text)

where $H_n = \tau_f/(\gamma S)$ is the 'normal depth' of the flow. The resulting flow front profile is a logarithmic curve shown in a dimensionless form by the thick dashed line in Figure 5.

Similar integration of Equation 2 for the case of a laminar (Poiseuille) flow gives:

$$\frac{xS}{H_n} = \frac{1}{2}\ln\left|\frac{z+1}{z-1}\right| - z \tag{6}$$

where $z = H/H_n$ and $H_n = [(3\mu V)/(\gamma S)]^{1/2}$, μ being the dynamic viscosity of the fluid. The resulting curve is shown by a solid line in Figure 5.

For Bingham flow, the shape of the front falls between the limits set by the plastic flow and the laminar flow, depending on a factor $N = (\tau_b)/(\gamma H_n)$, where τ_b is the Bingham yield strength. Three profiles are shown in Figure 5, for N equal to 0·25, 0·50 and 0·75.

For a frictional flow, the front is wedge-shaped and there is no normal depth. The wedge rises from the bed at an angle approximately equal to the arithmetic difference between the bulk friction angle (including any pore pressure effects) and the bed slope angle.

While these equations account for the characteristic shapes of some flow fronts (cf. Johnson, 1970, p. 453), they cannot be used to define the steady-state peak discharge. In fact, *for a homogeneous material*, there can be no surge tail in the steady state. To be constant in shape, the front would need to be supplied by a steady inflow of material from upstream and its constant maximum depth would be the normal depth corresponding to the supplied discharge rate. Such a behaviour is not characteristic of coarse debris flows and this in itself serves as evidence that surge flow is not homogeneous.

Heterogeneous surges: a model involving two materials

Equation 4 can be used to derive the depth profiles for heterogeneous surges in which the rheological relationship used to determine T varies with x. The following example is based on assumptions used in a simpler model by Hungr (1981, p. 290): it is assumed that a surge of viscous (Newtonian) slurry is supplied by material from upstream at a rate sufficient to maintain a normal depth H_n. The surge front contains a boulder accumulation which extends from the tip to a distance L behind it, measured along the slope using the coordinate system of Figure 4.

The concentration of large clasts decreases from a maximum at the tip to zero at the end of the frontal boulder accumulation. As a result, the frictional component of the flow resistance varies linearly from a maximum at $x = 0$ to zero at the end of the frontal accumulation ($x = L$). Such a variation in effective stress

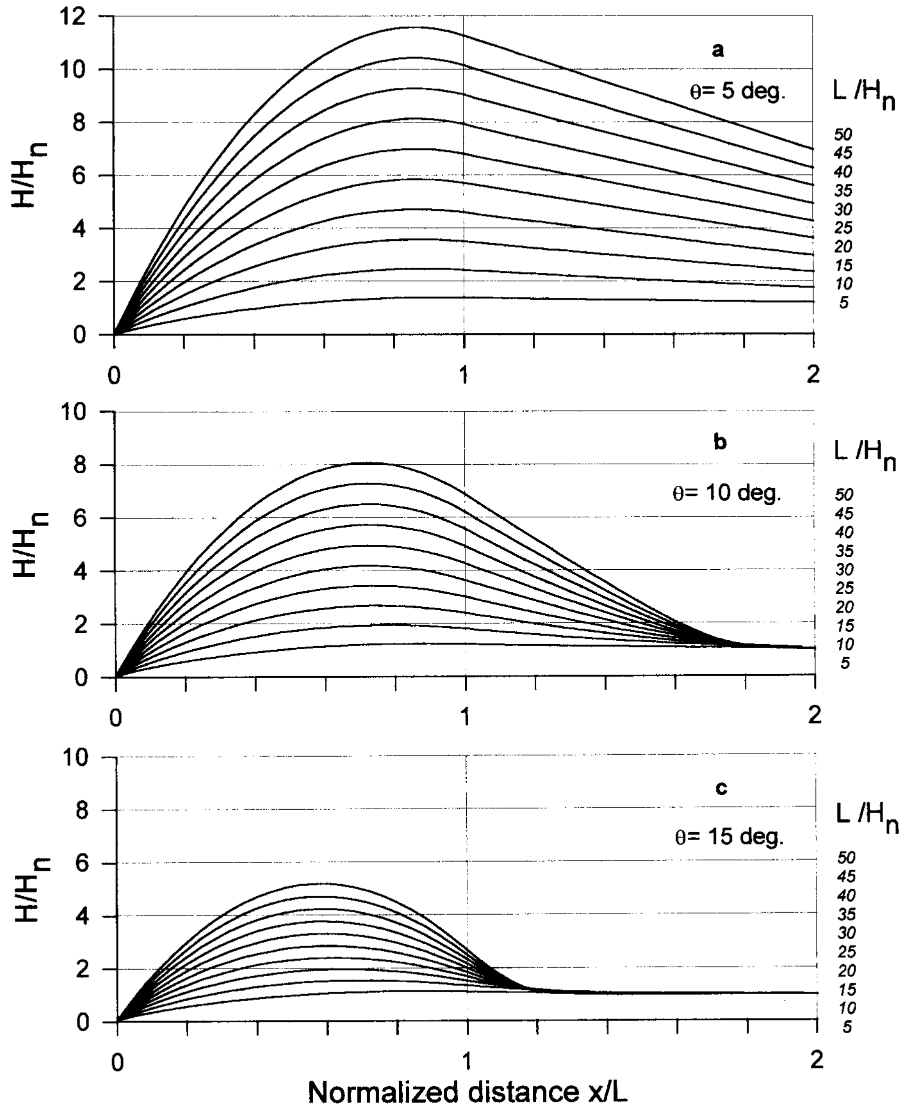

Figure 6. Dimensionless depth profiles of surges in a material comprising a frictional boulder front extending over a length L, followed by a viscous fluid with constant viscosity. H is flow depth and H_n is the normal depth. The profiles are drawn for three values of bed slope angle, θ, and 10 values of ratio L/H_n, as indicated

was measured in flume experiments and used in an unsteady flow model by Iverson (1997a).

$$T_f = \rho g H \cos\theta (1 - x\frac{r_u}{L}) \tan\phi \tag{7}$$

where ϕ is the dynamic friction angle of the boulder concentration (assumed as 32°). The pore pressure ratio, r_u, varies from zero at $x = 0$ to 1·0 at $x = L$. Thus, the flow is fully liquefied at the end of the boulder accumulation ($x = L$) and the frictional stresses disappear.

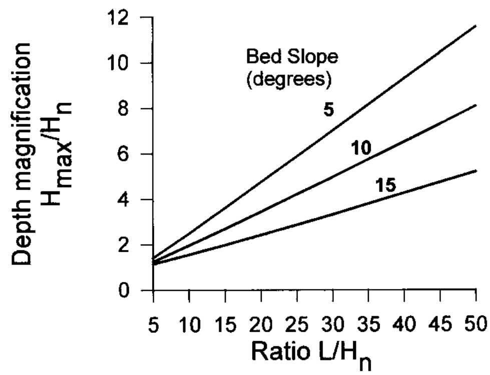

Figure 7. Magnification of the peak flow depth and discharge, as a function of the dimensionless length of the boulder front and the bed slope angle, derived from Figure 6

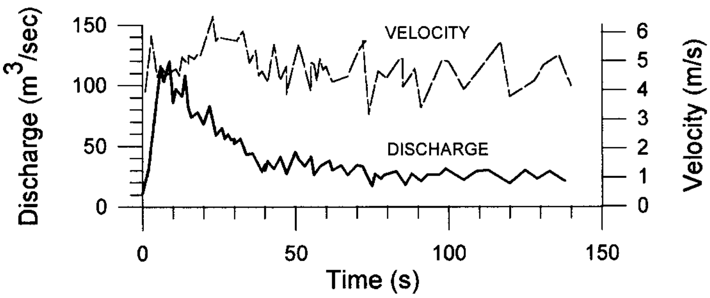

Figure 8. A record of a steady-state coarse debris flow surge, obtained using a spatial filter speedometer (Okuda *et al.*, 1980)

While the frictional stresses diminish with x, the slurry matrix content increases simultaneously and the viscous component of the resisting stress grows with it:

$$T_v = \frac{3V\mu}{H} \times \frac{x}{L} \tag{8}$$

where V is the mean velocity of the surge and μ the dynamic viscosity of the slurry. The viscous stress equals zero at $x = 0$ and increases to a maximum corresponding to a fully developed laminar (Poiseuille) flow at $x = L$. The same constant-viscosity, laminar flow is assumed to persist at all points upstream of the boulder front.

As suggested by Iverson (1997a), the net resisting stress in the frontal region can be obtained as the sum of the frictional and viscous components:

$$T = T_f + T_v \tag{9}$$

After substitution for T from Equation 9, Equation 4 was solved numerically, using the Runge–Kutta method. The results are shown in a dimensionless form, independent of viscosity, in Figure 6 for three slope angles (5, 10 and 15°) and a variety of values for the length of the boulder front, L. It is evident that the existence of a frictional front magnifies the peak depth of the surge far above the normal depth of the retained fluid. The amount of magnification depends on the bed slope and the length of the boulder front, relative to the normal depth of the slurry flow (Figure 7). For the range of input parameters used, the peak depth (and discharge) is magnified 2 to 12 times.

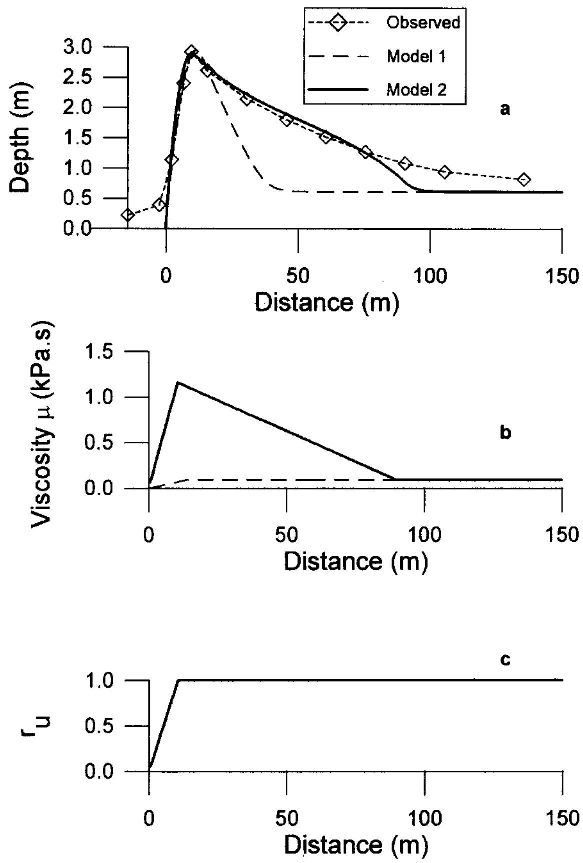

Figure 9. Analysis of data derived from Figure 8, using a numerical solution of Equation 4. To obtain the flow depth variation, the discharges of Figure 8 were divided by the mean velocity (3 m s^{-1}) and flow width (13 m). (a) Observed flow profile (diamond symbols), compared with predictions of Model 1 (uniform viscosity) and Model 2 (decreasing viscosity). (b) Dynamic viscosity used in Model 1 (dashed line) and Model 2 (full line). (c) Pore pressure ratio used in both models

This parametric study would produce an exactly equivalent result, if the material retained behind the boulder front was turbulent or had Bingham or other rheology. The shape of the profiles is determined not by the rheology of the retained material (as long as it is constant), but by the contrast between its flow resistance and that of the frictional front.

Comparison with measurements: variable viscosity model

An example of a surge profile in a coarse-grained debris flow is shown in Figure 8, which is a detailed observation of a moving debris flow at the Kamikamihori debris flow research station in the Japanese Alps (Okuda *et al.*, 1980). The surge exhibits an approximately constant surface velocity of 4·5 m s^{-1}, corresponding to a mean velocity of 3 m s^{-1} on the assumption of a laminar velocity profile. Yet its discharge is seen to vary between 30 and 120 m^2 s^{-1}, as a result of changing depth. The nearly constant velocity implies that the longitudinal shape of the surge changes little with time and that a certain steady-state equilibrium has indeed been approximately attained. The front of the surge is characteristically steep and its peak discharge short-lasting.

Figure 9 is a simulation of the observed behaviour using the model described above. Based on information provided by Okuda *et al.* (1980), the bed slope angle is 6·1° and the surface width of the flow is approximately 13 m. The mean unit weight of the debris was assumed as 20 kN m^{-3}. To convert the discharges of Figure 8

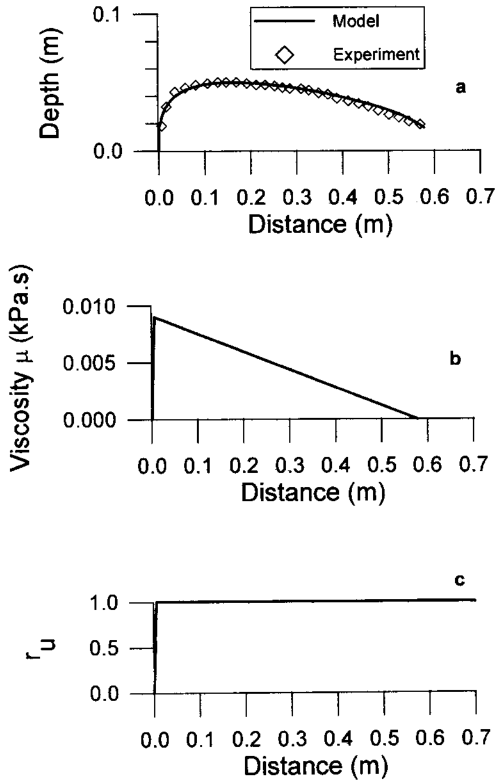

Figure 10. Analysis of a steady-state surge set up in a conveyor belt apparatus (Davies, 1990). (a) Comparison of measured (diamond symbols) and calculated (full line) flow profile. (b) Assumed distribution of dynamic viscosity. (c) Assumed distribution of the pore pressure ratio. The mean velocity was 0·39 m s^{-1} and the slope angle was 15°

into flow depths, they were divided by the flow width and mean velocity (13 m and 3 m s^{-1} respectively). The resulting depth profile is shown by the diamond symbols and the short-dashed line in Figure 9a.

The long-dashed line in the same figure (Model 1) is derived by interpolation of the dimensionless profiles from Figure 6 for a normal depth, H_n, of 0·615 m, a length of the boulder front, L, of 13·5 m and a slope angle of 6·1°, resulting in a depth magnification ratio of 5·0. The dynamic viscosity of the flow at the normal depth is 0·09 kPa s. The simulated depth profile has been superimposed on the observed so as to make the two coincident at the peak. The two-material model can match the depth magnification exhibited by the surge. However, the observed post-peak flow profile decreases much more gradually than predicted. To simulate this behaviour, it is necessary to introduce a linear decrease of viscosity over a length of 90 m, as shown in Figure 9b. The viscosity reduction rate equals 0·012 kPa s per m of length, from a maximum of 1·17 kPa s at the peak to the normal depth value at the end.

The resulting flow profile predicted by a numerical solution of Equation 4 is shown by the full line in Figure 9a (Model 2). The general trend of depth decrease is simulated well, although the slight convexity of the upstream end of the surge profile indicates that the viscosity reduction eventually ceases to be linear. The reduction is consistent with a gradual increase in water content along the length of the surge. It is easily attainable in view of the strong dependence of viscosity on solids concentration in dense slurries, as shown in Figure 3.

Another example is a steady-state surge set up in a laboratory conveyor belt apparatus by Davies (1990), using a mixture of PVC beads in water. The mean unit weight of the mixture was approximately 12 kN m^{-3} and the bed slope is 15°. The observed surge depth profile is shown by the diamond symbols in Figure 10a. The calculated depth profile, shown by the full line, is based on a frictional front 0·2 m long, followed by a viscous slurry. The slurry properties were determined by trial and error as an initial dynamic viscosity of

0·009 kPa s and a reduction rate of 0·016 kPa s per m (Figure 10b). The reduction rate is so high in this small-scale experiment that the normal depth is never reached and the surge tail approaches inviscid flow of zero thickness, probably accompanied by turbulence. As in the previous case, the requisite viscosity reduction rate is by no means unrealistic and would result from modest changes in solids concentration along the length of the surge.

CONCLUSIONS

The uniformly progressive flow model provides a simple tool for modelling the dynamic behaviour of steady-state debris flow surges. The nature of longitudinal sorting is as important in controlling the profile of a debris flow surge as is the rate of supply and the rheology of the material. Linear variations of flow resistance have produced good results in two examples. But there is no reason to expect that linearity will be universal. The variation is likely to depend on a variety of factors such as the bulk composition of the debris sources, the temporal and spatial variations in gradation and water content, the maturity of the longitudinal sorting and the efficacy of processes such as partial deposition of coarse clasts in boulder levees and distributary lobes.

The proposed model cannot yet be used for prediction, for lack of input data. However, several useful qualitative conclusions can be made on the basis of the above results.

(1) A substantial change in the peak discharge of a surge may result from changes in the volume of the boulder front. This may increase with time as the longitudinal sorting matures, or decrease suddenly if a part of the front is lost by deposition of a boulder levee at a widening of the channel.
(2) The steady-state peak discharge is a strong function of the slope angle. As shown in Figure 7, the greatest discharge (depth) magnification results where the slopes are flat.
(3) The steady-state surge is shorter on steep slopes and longer on flatter slopes, as the slurry backs up behind the front (Figure 6).
(4) The profile of the surge tail depends on the rate of decrease in solids concentration behind the peak.
(5) Conditions approximating steady uniform flow exist only in the vicinity of the surge peak, where $S_f = S$ and the flow surface is close to parallel with the bed (cf. Hungr *et al.*, 1984, p. 671). The rheology of the material present at this point is, however, variable. As can be observed in Figure 6, the peak region appears predominantly viscous on flat slopes, where it lies close to the end of the frontal accumulation. It is viscous-frictional on steeper slopes, as the peak region moves closer to the tip of the surge. It is almost fully frictional on very steep slopes (not shown in Figure 6). This has bearing on any attempts to derive the flow rheology from stage–discharge observations of peak flows as marked by mudlines in the field.

These general findings are independent of the rheology of the material or flow regime. Qualitatively, a discharge magnification effect will always result if there is a resistant front restraining more mobile fluid, whether Newtonian or non-Newtonian, laminar or turbulent.

To succeed in quantitative modelling of debris flows, we must study the process of longitudinal sorting and, particularly, learn to estimate the length of the boulder front. The necessary data will have to come from observations of full-scale debris flows in the field, or from carefully designed scaled experiments calibrated against field observations (e.g. Davies, 1990; Iverson, 1997a). Excessive reliance on laboratory-derived rheological constitutive relationships is not advisable. Models should be non-homogeneous and able to incorporate zones of contrasting rheology. These considerations do not apply to debris avalanches, whose short travel and limited saturation preclude the development of longitudinal sorting.

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